Global Volatility Shocks and the PPP Puzzle

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[PRELIMINARY WORK[1]]

Abstract

Most of the discussion about the Purchasing Power Parity (PPP) Puzzle of Rogoff (1996) has pertained to the reversion speed of deviations from PPP. Much less attention, however, has been given to the other component of the puzzle: the high volatilities of real exchange rates. In this paper, we provide a framework that is capable of explaining the econometric sources of these volatilities. First, we study the drivers of real exchange rate volatilities using a Cross-Sectionally Augmented Autoregressive Distributed Lag (CR-ARDL) panel framework and the conditional covariance matrices of the system with nominal exchange rates and price differentials. This analysis indicates that, for both emerging and developed markets, common factors are the main drivers of volatility. With this result in hand, we propose a novel econometric framework – based on the endogenous common volatility shocks model of Engle & Campos-Martins (2020) – that explains the sources of these volatilities as common second moment shocks. This framework allows us to give structure to the origins of these high volatilities and propose an extension to study their macro-financial drivers.

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1 Introduction

Since the breakdown of the Bretton Woods system, major shifts in the global economy and financial markets have exacerbated the magnitude of exchange rate fluctuations. While Friedman (1953) notoriously argued that exchange rate volatility is a manifestation of macroeconomic volatility, empirical studies have uncovered a range of anomalies and puzzles that contradict theoretical models of exchange rates. Amongst the many unanswered questions raised by the empirical international finance literature, one of the most persist ones has been the Purchasing Power Parity (PPP) Puzzle of Rogoff (1996).

In his words, the puzzle presented by Kenneth Rogoff in his seminal 1996 paper is: "How can we reconcile the enormous short-term volatility of real exchange rates with the extremely slow rate at which shocks appear to damp out?". The very slow[2] speed of adjustment of shocks to real exchange rates has been the source of considerable theoretical and empirical research, with relative success (Taylor (2002)). From the empirical side, Cheung & Lai (2000) argue that the persistence of these deviations from PPP is mainly due to the non-linearity of the adjustment process. From the theory side, Carvalho & Nechio (2011) propose a model that is capable of generating the persistence observed in the data by introducing heterogeneity in the frequency of price changes across sectors.

Most of the discussion about the PPP Puzzle has, however, only pertained to the reversion speed of deviations from PPP. As Taylor (2002) argues, much less attention has been given to the other component of the puzzle: the high short-term volatilities of real exchange rates. Even if we consider that the recent literature has established that exchange rates do revert to the PPP equilibrium rate over the medium term at a speed that is consistent with theory, the volatilities present in the data in the short-run, at least under floating regimes, still remain a puzzle. According to Ganguly & Breuer (2010), this piece of the puzzle is arguably more important to understand than the first because of its implications for trade, investment, and economic growth.

Mussa (1986) was the first piece of research to analyse second moments of real exchange rates with a focus on short-run fluctuations. The author calculates unconditional variances and covariances for real exchange rates, nominal exchange rates, and price differentials for fixed and flexible exchange rate periods. This exercise shows that not only these variances and covariances changed from one regime to the other, but real exchange rates present significantly higher variances under flexible regimes. This finding surprised the field, as theories going back to Friedman (1953) maintain that a flexible exchange rate should be useful as an alternative adjustment mechanism of relative prices when nominal prices are not free to adjust. The critique of Mussa (1986) was, and still is, extremely influential, changing the course of exchange rate models. More recently, Taylor (2002) updated the analysis of Mussa (1986) by allowing slowly evolving

[2]The literature half-life estimates for real exchange rates are generally between 3 and 6 years.

deterministic trends and studying deviations from these trends. The author finds important quantitative differences in the residual variances with floating regimes exhibiting much larger shocks to the real exchange rate process, accounting for the significantly larger deviations from PPP in these eras. Ganguly & Breuer (2010) also explore the short-run volatility of real exchange rates. The authors conduct a simple unconditional variance decomposition of real exchange rate into nominal exchange rate volatility and relative price volatility, after controlling for real and nominal factors. Finally, Bergin et al. (2014) study changes in variances and covariances between different periods using simulations from a Vector Error Correction framework of Cheung et al. (2004).

Although the literature presented above has achieved important conclusions regarding the short-run volatilities of real exchange rates during different regimes, it has not been able to answer more meaningful questions about these short-run dynamics. As, for instance, what drives these high and persistence variations. Moreover, in all studies, the analyses focus on calculating realised unconditional volatilities and covariances for different periods and drawing inference from their differences. Even though this might be useful to superficially understand the differences in unconditional second moments between different currency regimes, it is certainly not the most recommended econometric framework to study the dynamics of short-run volatilities.

In this paper, we use latest develops from the financial econometrics literature in modelling second moments dynamics to study this remaining part of the PPP Puzzle. In a first step of the analysis, we present an econometric framework based on the Cross-Sectionally Augmented Autoregressive Distributed Lag (CR-ARDL) model of Pesaran (2006) for the decomposition of real exchange rate volatility into its building blocks. The results from our proposed framework indicate that the most important driver of real exchange rate volatilities are the common dynamics (considered as group means in our framework). This suggests that a key part of understanding the remaining part of the PPP Puzzle – that is understanding the high short-to-medium-term volatilities of real exchange rates – is modelling the cross-sectional correlations in real exchange rate volatilities.

Inspired by the importance of cross-sectional correlations in explaining real exchange rate volatilities, we propose an econometric model based on the endogenous common volatility shocks framework of Engle & Campos-Martins (2020) to model the dynamics of real exchange rate volatilities in a second step of this study. This framework presents encouraging results when modelling the aforementioned cross-sectional correlations. It successfully gives structure to these common volatility dependencies in real exchange rates and further allows us to propose an extension to the framework in order to study the impact of other macro-financial variables on this common volatility movements. In fact, this extension is a general framework and can be considered as an extension of the model of Engle & Campos-Martins (2020) to allow for exogenous drivers of common volatility shocks. The results from our proposed extension show common shocks to interest rate differentials as a key driver of common volatility shocks in exchange rates, hence building a bridge between our analysis of the PPP Puzzle and the Interest Rate Parity literature.

This research relates to the empirical international finance literature and, specifically, to the study of volatility of real exchange rates. By using latest developments in second moments modelling to study the dynamics and sources of real exchange rate volatility, we expect to shed some light on the the remaining part of the PPP Puzzle. In a brother sense, the results from this study can also be seen as a motivation for applications of the endogenous common volatility shocks framework of Engle & Campos-Martins (2020) and our extension to exogenous drivers to other asset classes. Within the next pages, Section 2 presents a brief literature review on the PPP Puzzle and the analysis of real exchange rate volatility. Section 3 describes the dataset used for this research, the transformations required and some preliminary results. In Section 4, we present the econometric framework used for estimating second moments and decomposing real exchange rate volatility into its building components. Based on the results from Section 4, Section 5 presents a model that gives structure to the common volatility shocks to exchange rates and expands this model to allow for exogenous drivers of these common shocks. Section 6 concludes by linking our findings regarding the PPP Puzzle to other topics in the empirical international finance literature.

2 Literature Review

2.1 The PPP Puzzle

Purchasing Power Parity (PPP) is the disarmingly simple empirical proposition that, once converted to a common currency, national price levels should be equal. It was was articulated by scholars of the Salamnca school in the sixteenth century in Spain but first proposed by Swedish economist Gustav Cassel (Cassel (1921) and Cassel (1922)) as mean for setting relative gold parities in exchange rates after World War I. Though PPP had been discussed previously by classical economists such as John Stuart Mill, Alfred Marshall, and Ludwig von Mises, Cassel was really the first to treat PPP as a practical empirical theory.

The basic idea is that if the goods market arbitrage enforces broad parity in prices across a sufficient range of goods via law of one price, then, by construction, there should also be a high correlation in aggregate price levels. Some might say that, given the observed volatilities in exchange rates and differences in prices of the same good across the world, the PPP is only a theoretical construct that does not apply in practice. Nevertheless, "while a few empirically literate economists take PPP seriously as a short-term proposition, most instinctively believe in some variant of PPP as an anchor for long-run real exchange rates" Rogoff (1996).

Empirical support for PPP has changed over the years. From a historical standpoint, there have been numerous studies of PPP with various datasets[3]. McCloskey & Zecher (1984) argue that PPP worked very well under the gold standard before 1914. Diebold et al. (1991) explore a very long run panel of nineteenth-century data for six countries and find support for PPP based on the low-frequency information lacking in short-sample studies. Abuaf & Jorion (1990) study a century of Dollar-Franc-Sterling exchange rate data and verified PPP. Lothian & Taylor (1996) further confirm the results from Abuaf & Jorion (1990) using two centuries of Dollar-Franc-Sterling. Lothian (1990) also finds evidence that real exchange rates were stationary in Japan, the US, the UK and France for the period 1975-1986. More recently, Engel et al. (2015) and Ca'Zorzi et al. (2020) find that PPP based forecasts for exchange rates have the best out-of-sample performance from all models considered.

By the late 90s, the empirical international finance literature had arrived at a surprising degree of consensus over some basic facts regarding exchange rates. First, a number of studies had presented evidence that points towards a PPP equilibrium of exchange rates in the long-run. Second, that short-run deviations from PPP are large and volatile. Puzzled by this empirical dichotomy, Rogoff (1996) proposed the following PPP Puzzle: How can one reconcile the enormous short-term volatility of real exchange rates with the extremely slow rate at which shocks appear to damp? The most obvious explanation for the short-run volatility of real exchange rates would be price stickness. This is the essence of the Dornbusch (1976) overshooting model of nominal and real exchange rate volatility. Consensus estimates for the rate at which PPP deviations damp, however, suggest a half-life of three to six years, seemingly far too long to be explained by nominal rigidities.

The puzzle proposed by Rogoff (1996) created a new sub-field within the international finance literature, and inspired countless papers in both the theory and empirics of PPP. In one of the first attempts to solve the puzzle, Clarida & Gali (1994) and Rogers (1999) identify the relevance of multiple shocks in explaining the variability of real exchange rates, but their results still do not resolve the PPP Puzzle. The first meaningful progress in "solving" the PPP Puzzle is the work of Cheung & Lai (2000). Using impulse response analysis, Cheung & Lai (2000) analyse the adjustment dynamics of real exchange rates by evaluating both the sample and half-life measure and its estimation accuracy. The impulse response analysis shows that the shocks impact tends to amplify first before it dissipates. The full impact of the shock is not felt immediately but until a few periods after the initial shock. Hence, following the shock, the real exchange rate does not revert to its long-run value monotonically, but in hump-shaped manner. Cheung & Lai (2000) find that this non-monotonic adjustment contributes considerably to generate persistency in real exchange rates.

[3]For a full review of the literature up to the 90s, one can refer to Froot & Rogoff (1995).

In a following paper, Cheung et al. (2004) present additional evidence on the convergence speeds of nominal exchange rates and prices. Using Vector Error Correction (VEC) analysis, the authors estimate the speeds at which the individual variables revert to their long-run values. The VEC analysis provides an alternative, easier way to measure those convergences speeds than the previous state-space studies (as Engel & Morley (2001)). While taking a different approach, the results from Cheung et al. (2004) corroborate those of Engel & Morley (2001) that nominal exchange rates do converge to at a much slower rate than prices. Half-lives of nominal exchange rates are estimated to be from 3 to 6 years, whereas half-lives of prices are found to be substantially shorter (mostly abut 1 to 2 years). Cheung et al. (2004) also show that about 60% to 90% of PPP disequilibrium adjustment takes place through nominal exchange rate adjustment. Hence, it is mostly nominal exchange rate adjustment – not price adjustment – that drives real exchange rates towards parity. As such, the observed rate of PPP reversion reflects primarily the speed of nominal exchange rate convergence. Should nominal exchange rates converge much more slowly than prices, the PPP reversion speed can be slower than the price convergence speed, as described by the PPP puzzle.

Trying to further address the PPP Puzzle, Taylor (2002) recreates the analysis of Mussa (1986) with empirical innovations by controlling for long-run deviations from PPP – Balassa (1964) and Samuelson (1964) like effects – and using longer span of historical data. When investigating four different currency regimes, Taylor (2002) finds important differences in the residual variance, with the floating regimes exhibiting much larger shocks to the real exchange rate process accounting for the much larger deviations from PPP during these eras. According to the author, these results show that there was relatively little change in the ability of international market integration to smooth out real exchange rate shocks. Instead, Taylor (2002) argues, the changes in the variance of the shocks reinforce the conclusion of Mussa (1986) of seeing exchange rate regimes as a major determinant of real exchange rate behaviour. More importantly, the author concludes that changes in the persistence of the process play little role in explaining why the behaviour of real exchange rates changes so much from one regime to the other. "Changes in the volatility of the shocks explain virtually all changes in the volatility in the real exchange rate across space and time" (Taylor (2002)). Therefore, understanding the dynamics and sources of these shocks is crucial for a better understanding of the PPP Puzzle. In fact, Taylor (2002) defends that "further study will be needed to incorporate these dynamics into an econometric PPP model and measure them in historical and contemporary samples".

As suggested by Cheung & Lai (2000), one approach to resolving part the PPP Puzzle of Rogoff (1996) lies in allowing for nonlinear dynamics in real exchange rate adjustment. More recently, however, a contribution from the theory side has also been able to solve the persistency part of the puzzle. Carvalho & Nechio (2011) study the PPP Puzzle in a multisector, two-country, sticky-price model. In their model, sectors differ in the extend of price stickiness, leading to heterogeneous sectoral real exchange rate dynamics. By introducing heterogeneity in the frequency of price changes across sectors, Carvalho & Nechio (2011) are capable of generating the persistence in deviations from PPP as observed in the data.

2.2 The remaining PPP Puzzle: High short-term volatilities

Empirical work that focuses on understanding the dynamics of real exchange rates and the PPP Puzzle must grapple with the two key properties: the reversion speed of deviations from PPP and the high short term volatility of the disturbance term. Most of the discussion of the literature up to this point has pertained to the reversion speed, which is a medium-to-long-term phenomenon. Nevertheless, the literature also acknowledges that more attention should be given to the other part of the puzzle. As Taylor & Taylor (2004) put it "even if the current work can establish that exchange rates do revert to the PPP rate over the medium term at a more reasonable speed, the volatilities present in the data in the short-run, at least under floating regimes, still cause considerable mystification".

While much work has been done directed at the first piece of Rogoff (1996) PPP Puzzle – studying the speed of convergence of deviations from PPP equilibrium – Ganguly & Breuer (2010) also defend that the high short-to-medium-term volatility piece is arguably more important to understand because of its implications for trade, investment, and economic growth. Yet, "real exchange rate volatility has received sporadic attention[4], at best" Ganguly & Breuer (2010).

Ganguly & Breuer (2010) build on the work of Hausmann et al. (2006), who find that the volatility of real exchange rates in developing markets is 2.5 times higher than for industrialized countries, even when controlling for real shocks. Like the model of Hausmann et al. (2006), Ganguly & Breuer (2010) include real factors but also includes domestic and external monetary and financial factors and trade balances. With the aim of better understanding the reasons for the high volatilities of real exchange rates, Ganguly & Breuer (2010) also conduct a simple variance decomposition of the real exchange rate, after controlling for real and nominal factors. This decomposition of the residual variance allow the authors to calculate the contributions of unexplained nominal exchange rate volatility, unexplained relative price volatility and their covariance to the residual proportion of real exchange rate volatility.

The analysis of Ganguly & Breuer (2010) produces three main findings. With the inclusion of nominal factors, their model substantially reduces the real exchange rate volatility spread between developing and developed economies, hence helping to explain

[4]Contributions include Edwards (1987), Côté (1994), Hausmann & Gavin (1996), McKenzie (1999), Hau (2000), Hau (2002), Clark et al. (2004), and Hausmann et al. (2006), Morales-Zumaquero & Sosvilla-Rivero (2010), and Cevik et al. (2017).

Hausmann et al. (2006) finding. The authors also find evidence that nominal factors matter in both the short and long-run. Nominal factors can have long-lived effect on the volatility of real exchange rate. Ganguly & Breuer (2010) also find that for developing countries, a much larger share of real exchange rate volatility stems from relative price than for industrial countries. This finding persists in both the short and the long-run.

Bergin et al. (2014) develop an updated version of the Mussa (1986) critique. The authors ask whether recent findings regarding dynamics of real exchange rate studying the standard post-Bretton Woods dataset apply also to the Bretton Woods period of generally fixed exchange rates. Specifically, the method of Pesaran (2006) is adapted to estimate an autoregression of the real exchange rate over the Bretton Woods and post-Bretton Woods periods for a panel of 20 industrialized countries. In addition, the authores estimate a two-equation Vector Error Correction Model (VECM) to decompose the real exchange rate into its nominal exchange rate and relative price components.

The key finding of Bergin et al. (2014) is that the dynamic properties of the real exchange rate differ between these two periods, in accordance with the original results from Mussa (1986). The methodology of Bergin et al. (2014) for decomposing real exchange rate changes into their underlying components is closely related to Cheung et al. (2004), but the latter are interested only in the flexible exchange rate period and do not implement panel techniques.

Overall, the empirical international finance literature has achieved a fair consensus that some sort of PPP equilibrium holds in the long-run. Moreover, both advances from the empirical side and the theory side have addressed why deviations from this PPP equilibrium might be so persistent. As indicated by Taylor (2002), Taylor & Taylor (2004), and Bergin et al. (2014), a more important and interesting question regarding the PPP Puzzle that still remains unanswered is why real exchange rates are so volatile in the short-run. In this paper, we apply late developments from the financial econometrics literature to study the drivers of real exchange rate volatility and propose a novel econometric framework that is capable of explaining the sources of these volatilities as common second moment shocks.

We divide this analysis into two steps. In the first one, we propose a panel model for the decomposition of real exchange rate volatility into its building components. This decomposition allows us to analyse the importance of each of the components and serves as a guideline to the model proposed in the following section. Inspired by the results from step one, we propose an econometric framework based on the work of Engle & Campos-Martins (2020) that is able to model the origins of the short-term volatility in real exchange rates. This framework further allows us to give structures to these volatilities and study their macro-financial drivers.

3 Data and Transformations

The main object of our study is the real exchange rate series for a multiplicity of countries. In order to obtain a set of countries which is representative for both emerging and developed markets but, at the same time, only selects relevant currencies with enough liquidity, we follow the methodology of BIS (2019) and select the thirty most traded currencies in the world. A list with the full set of currencies and their market classification according to the MSCI (2020) Emerging-Developed Market classification can be found in Appendix A.

Because we need both nominal exchange rates and price series in order to construct the series for the real exchange rates, we consider the data at a monthly frequency. This is the highest frequency for the price series and usually what is referred to when analysing the short-run behaviour of real exchange rates. The series for CPI and nominal exchange rates (period mean) were extracted from the IMF International Financial Statistics for all countries considered in the study from Jan 1990 to June 2020. The CPI series are standardized to be unity at the most recent observation. This makes price levels comparable and allows for easy interpretation of real exchange rates. The nominal exchange rate series are considered as the home currency unit of one US Dollar. We decided to use a currency as base – rather than using trade weighted measures of real exchange rates – to directly evaluate the impact of changes in the nominal exchange rates and price differentials on the real exchange rates. We decided to use the US Dollar as base because most currencies are usually denominated in this base and this is the exchange rate in which most of the trading takes place[5].

With this data in hand, for each of the countries, we define the following series:

- Nominal exchange rates (USD base): $E_{i,t}$
- Price ratios (to USD): $\tilde{P}_{i,t} = \frac{P_{USD,t}}{P_{i,t}}$
- Real exchange rate (USD base): $R_{i,t} = E_{i,t}\tilde{P}_{i,t}$

where i represents the country indicator for each of the countries considered in our sample and t stands for the month indicator from January 1990 to June 2020.

By constructing the real exchange rate in the above manner, we keep the standard procedure from the literature. Moreover, the base one in the latest observation of the price ratio allows us to calculate the level of real exchange that is comparable to the latest value of the nominal exchange rate. Figure 1 plots these three series for TRY, the

[5] As a robustness check, we have also estimated our models with other currencies as base in order to guarantee results were not being driven by changes in the base currency. These results are available on request.

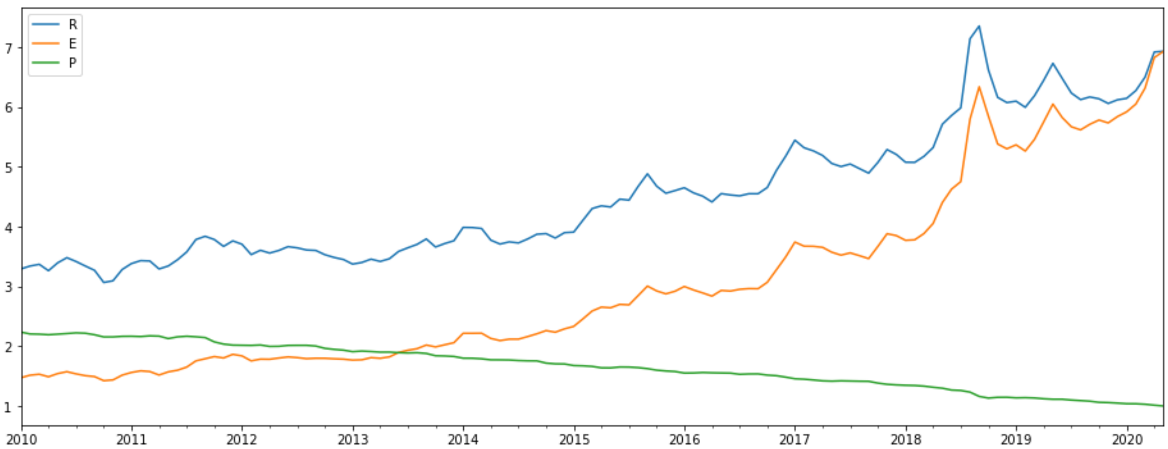

Figure 1: Real exchange rate ($R_{i,t}$), nominal exchange rate ($E_{i,t}$) and price ratio ($\tilde{P}_{i,t}$) for the Turkish Lira (TRY) since 2010.

Turkish Lira. By looking at Figure 1, one can see what would be the equivalent real exchange rate at today's prices and compare it with the nominal exchange rate at the time.

As it was noted by Mussa (1986) and we can also see in Figure 1, both exchange rate series are generally significantly more volatile than the price series in the short-to-medium-run, even for countries with considerable inflation. As a result, in order to control for this significant difference in the behaviour of prices and exchange rates, we focus on the standardized returns[6] of each of the series to study the second moments dynamics of real exchange rates. From now on, we will refer to $r^R_{i,t}$ as the standardized return of $R_{i,t}$, $r^E_{i,t}$ as the standardized return of $E_{i,t}$, and $r^P_{i,t}$ as the standardized return of $\tilde{P}_{i,t}$.

4 Decomposing Real Exchange Rate Volatility: A Panel Approach

Once we have the standardized series of returns for each of the variables we are interested in, we can proceed to the first step of our formal analysis. The objective of this first step is to develop an econometric framework to decompose the dynamics of real exchange rate volatility into its building blocks. We start with a description of the econometric methodology used to estimate second moments. We then introduce an econometric framework for the decomposition of real exchange rate volatility and present

[6] At this point, we simply standardize returns using full sample realised means and standard deviations.

the estimation results. These open the door for the model proposed in the second step of the analysis in Section 5.

4.1 Estimating conditional second moments

A crucial part of studying second moments of any given series is the methodology used to compute or estimate them. There are many ways of studying second moments. One can simply calculate the sample variances and covariances as realised second moments over some arbitrary time period, as introduced in modern econometrics by Andersen & Bollerslev (1998). Another approach when studying volatility is to use the implied volatility given a model for asset prices; as, for example, the VIX measure of volatility from CBOE (2009). Since we need a dynamic measure of second moments that is as agnostic as possible, we will use estimates of conditional second moments as introduced in the literature by Engle (1982) and Bollerslev (1986).

In this subsection we will present the methodologies used to estimate the conditional volatilities of real exchange rates and the conditional covariance matrices of the system with price differentials and nominal exchange rates. We base our methodology in the work of Cappiello et al. (2006) to select the best univariate and multivariate second moments models for global equities and bonds. As it was discussed in the previous section, the object of study here will be the standardized returns of each of these series.

4.1.1 Modelling real exchange rate volatilities

The first step is to build the series of conditional variances for the real exchange rate standardized returns. For a given country i, let $r_{i,t}^{R}$ be the standardized return of real exchange rates. Moreover, let $\mathcal{F}_{t}^{R}$ denote the sigma field generated by the past values of $r_{i,t}^{R}$. Following the approach of Engle (1982) and Bollerslev (1986), we can then write the conditional variance of $r_{i,t}^{R}$ as:

$$VAR[r_{i,t}^{R}|\mathcal{F}_{t-1}^{R}] = VAR_{t-1}[r_{i,t}^{R}] \equiv \sigma_{i,t}^{2,R} \tag{1}$$

which we define as $\sigma_{i,t}^{2,R}$ for easiness of notation.

As we want to keep our analysis as agnostic as possible regarding the model for $\sigma_{i,t}^{2,R}$, we follow the approach of Cappiello et al. (2006) and do an specification search on the following models:

- The TARCH of Zakoian (1994);
- The GJR-GARCH of Glosten et al. (1993);
- The EGARCH of Nelson (1991).

Allowing for up to two lags of each possible element in each of the specifications above. The best model is the chosen according to the Bayesian Information Criterion (BIC) of Schwarz et al. (1978). The model specification of each of these models used in the specification search can be found in Appendix B, as well as the model chosen for each of the real exchange rate series.

4.1.2 Second moments of prices and nominal exchange rates

The next step is to estimate the conditional second moments of the system with price differentials and nominal exchange rates. For a given country i, we begin by defining the following standardized return vector:

$$\mathbf{r}_t \equiv \begin{bmatrix} r^E_{i,t} \\ r^P_{i,t} \end{bmatrix} \tag{2}$$

where $r^E_{i,t}$ represents the standardized returns of nominal exchange rate and $r^P_{i,t}$ the standardized returns of price differentials.

As the components of $\mathbf{r}_t$ are standardized, we assume it to be a mean zero random vector. Moreover, let $\mathcal{F}^{EP}_{t-1}$ denote the sigma field generated by the past values of $\mathbf{r}_t$. If we assume that the conditional covariance matrix $\mathbf{\Sigma}_t$ is measurable with respect to $\mathcal{F}^{EP}_{t-1}$ and that $\mathbf{r}_t$ is conditionally normal[7], then the conditional distribution of $\mathbf{r}_t$ can be written as:

$$\mathbf{r}_t|\mathcal{F}^{EP}_{t-1} \sim N(0, \mathbf{\Sigma}_t) \tag{3}$$

where:

$$\mathbf{\Sigma}_t = \begin{bmatrix} VAR[r^E_{i,t}|\mathcal{F}^{EP}_{t-1}] & COV[r^E_{i,t}, r^P_{i,t}|\mathcal{F}^{EP}_{t-1}] \\ COV[r^E_{i,t}, r^P_{i,t}|\mathcal{F}^{EP}_{t-1}] & VAR[r^P_{i,t}|\mathcal{F}^{EP}_{t-1}] \end{bmatrix} \equiv \begin{bmatrix} \sigma^{2,E}_{i,t} & \sigma^{EP}_{i,t} \\ \sigma^{EP}_{i,t} & \sigma^{2,P}_{i,t} \end{bmatrix} \tag{4}$$

In this study, we estimate the components of $\mathbf{\Sigma}_t$ according to the Asymmetric Generalized Dynamic Conditional Correlation (AG-DCC) GARCH of Cappiello et al. (2006). As we did for the univariate volatility models for real exchange rates, we allow for up to two lags of each component of the model and choose the model for each country according to the BIC of Schwarz et al. (1978). The full details about the AG-DCC GARCH of Cappiello et al. (2006) can be found in Appendix C, as well as the structure of the AG-DCC GARCH model chosen for each country.

4.2 Decomposing real exchange rate volatility

We now turn to the decomposition of real exchange rate volatility into its building components. After estimating the models described in Section 4.1, we end up with the

[7] Standard assumptions of multivariate Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models.

following series for each of the i countries in our sample:

- $\hat{\sigma}^{2,R}_{i,t}$: Fitted real exchange conditional variance;
- $\hat{\sigma}^{2,E}_{i,t}$: Fitted nominal exchange rate conditional variance;
- $\hat{\sigma}^{2,P}_{i,t}$: Fitted price differentials conditional variance;
- $\hat{\sigma}^{EP}_{i,t}$: Fitted conditional covariance between nominal exchange rates and price differentials.

Recall that we have monthly series for 29 exchange rates and price differentials against the United States from January 1990 to June 2020. As a result, for each of the estimated series described above, we have a panel dataset with a small N (29) and a fairly large T (365). We can therefore use a panel econometric technique to perform the decomposition of real exchange rate volatility into its building components.

The most straightforward way to perform this decomposition is to consider the real exchange rate volatilities as the dependent variables and the variables from the system with price differentials and nominal exchange rates as explanatory variables. Because, by construction, these volatilities are correlated over time, we also need to account for their time dependencies in the model.

Auto-Regressive Distributed Lag (ARDL) models are standard least squares regressions that include lags of both the dependent variable and explanatory variables as regressors (Greene (2003)). In our setting, a standard ARDL model takes the form:

$$\sigma^{2,R}_{i,t} = \eta_i + \alpha_i \sigma^{2,R}_{i,t-1} + \beta_i \mathbf{x}_{i,t} + v_{i,t} \tag{5}$$

for i in $\{1, \ldots, N\}$ and t in $\{1, \ldots, T\}$ and where $\mathbf{x}_{i,t} = [\sigma^{2,E}_{i,t}, \sigma^{2,P}_{i,t}, \sigma^{EP}_{i,t}]^T$ and β_i is a vector of coefficients.

Nevertheless, the panel of real exchange rate volatilities has one very important property. As it will be extensively studied in Section 5, real exchange rate volatilities are significantly and positively correlated. This means that the residuals from Equation 5 will be cross sectionally correlated, hence violating the key assumption of cross-sectional independence from the ARDL model.

In order to address the cross sectional correlation in real exchange rate volatilities, we propose using the Cross-Sectionally Augmented Autoregressive Distributed Lag (CR-ARDL) model with the Common Correlated Effects Mean Groups (CCE-MG) estimator of Pesaran (2006) to study the decomposition of real exchange rate volatilities. Pesaran (2006) allows for a form of cross-sectional dependence by introducing an error component with a factor structure. The author shows that one can allow for the presence of this unobserved common factor with a heterogeneous loading parameter. We can control for the presence of this error component by augmenting the ARDL model above by including time-specific means as additional explanatory variables. That is, by estimating the following:

$$\sigma^{2R}_{i,t} = \eta_i + \alpha_i \sigma^{2R}_{i,t-1} + \beta_i \mathbf{x}_{i,t} + \gamma_i \bar{\sigma}^{2R}_{t-1} + \delta_i \bar{\mathbf{x}}_{i,t} + \omega_i \bar{\sigma}^{2R}_{t} + v_{i,t} \tag{6}$$

for i in $\{1, \ldots, N\}$ and t in $\{1, \ldots, T\}$ and where $\mathbf{x}_{i,t} = [\sigma^{2,E}_{i,t}, \sigma^{2,P}_{i,t}, \sigma^{EP}_{i,t}]^T$, the $^-$ stands for the group mean values and β_i and δ_i are vector of coefficients.

The parameters from the model described in Equation 6 are then estimated using the mean groups estimator of Pesaran & Smith (1995), generating the Common Correlated Effects Mean Groups (CCE-MG) estimator of Pesaran (2006). We perform the mean groups estimation considering all countries as one group as well as clustering the countries in groups according to the MSCI (2020) market classification into emerging and developed markets that we have been using throughout this paper.

The estimation results for the model from Equation 6 can be found in Table 1. By looking at Table 1, one can notice a few results. First, real exchange rate volatilities seem to more persistent in developed markets than in emerging markets. Moreover, as one would expect, nominal exchange rate volatilities are the most significant driver of real exchange rate volatilities from the covariance matrix of the system with price differentials and nominal exchange rates.

Although nominal exchange rate volatilities are shown to play a significant role in all specifications, the main results from Table 1 are regarding the (ω) coefficients for the simultaneous real exchange rate variance group mean ($\bar{\sigma}^{2,R}_t$). These estimation results indicate that not only is this term significant for all group mean specifications considered, but that the most important driver of real exchange rate volatilities are the group means. This result holds at almost all levels of significance and for all groups considered in our group means estimator.

As proposed by Taylor (2002), Taylor & Taylor (2004), Ganguly & Breuer (2010) and many others, understanding the drivers of real exchange rate volatilities in the short-to-medium-run is the key remaining part of the PPP Puzzle. Our estimation results from the decomposition proposed in Equation 6 show that this task can be reframed. According to the results presented in Table 1, if one seeks to understand the sources of high real exchange rate volatility – and hence understand the remaining part of the PPP Puzzle – one must address the cross sectional correlation in volatilities across currencies. In the next section, we propose an econometric framework that is capable of explaining this cross sectional correlation in volatilities as common volatility shocks. This framework allows us to give structure to the origins of these high volatilities and study their macro-financial drivers.

Table 1: Estimated coefficients and p-values from the Common Correlated Effects Mean Groups (CCE-MG) estimator for Cross-Sectionally Augmented Autoregressive Distributed Lag (CR-ARDL) model for $\sigma^{2,R}_{i,t}$.

	All	**EMs**	**DMs**
η	0.0197	0.0422	0.1625
(intercept)	(0.16)	(0.06)*	(0.06)*
α	0.3599	0.3114	0.3980
$(\sigma^{2,R}_{i,t-1})$	(0.02)**	(0.10)	(0.02)**
β_1	0.2553	0.2663	0.2017
$(\sigma^{EP}_{i,t})$	(0.13)	(0.09)*	(0.15)
β_2	0.1956	0.2027	0.1851
$(\sigma^{2,E}_{i,t})$	(0.08)*	(0.02)**	(0.04)**
β_3	0.0433	0.0575	0.0245
$(\sigma^{2,P}_{i,t})$	(0.09)*	(0.06)*	(0.09)*
γ	-0.3529	-0.3642	-0.1114
$(\bar{\sigma}^{2,R}_{t-1})$	(0.17)	(0.15)	(0.16)
δ_1	-0.2099	-0.4258	0.1703
$(\bar{\sigma}^{EP}_{t})$	(0.30)	(0.31)	(0.25)
δ_2	-0.1233	-0.1758	-0.0214
$(\bar{\sigma}^{2,E}_{t})$	(0.30)	(0.24)	(0.46)
δ_3	-0.0905	-0.1163	-0.0039
$(\bar{\sigma}^{2,P}_{t})$	(0.28)	(0.35)	(0.34)
ω	0.8950	1.0564	0.3240
$(\bar{\sigma}^{2,R}_{t})$	(0.04)**	(0.04)**	(0.03)**

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

5 A Model for Volatility Co-Movements

The results from the previous section show that a key part of understanding the remaining part of the PPP Puzzle – that is understanding the high short-to-medium-term volatilities of real exchange rates – is understanding the cross-sectional correlations in real exchange rate volatilities. We begin this section by presenting the endogenous model of volatility co-movements introduced by Engle & Campos-Martins (2020) to address the cross-sectional correlation in real exchange rate volatilities. We conclude by expanding the so-called GEOVOL model to allow for exogenous variables when estimating common volatility shocks. This allows us to study the influence of other macro-financial variables on the volatility co-movements of real exchange rates.

5.1 Volatility co-movements and shocks to volatilities

The standard asset pricing model can be formulated for $NX1$ vector of returns $\mathbf{r}_t \equiv (r_{1,t}, \ldots, r_{N,t})$ as:

$$\mathbf{f}_t = \mathbf{w}'_{t-1}\mathbf{r}_t \tag{7}$$

$$\mathbf{r}_t = r^f + \beta\mathbf{f}_t + diag\{\sqrt{\mathbf{h_t}}\}\mathbf{e}_t \tag{8}$$

If factors are sufficient to reduce contemporaneous cross-sectional correlations in $\mathbf{e}_t$, then we have that:

$$\mathbb{E}_{t-1}(\boldsymbol{e}_t\boldsymbol{e}'_t) = \mathbb{I} \tag{9}$$

This means that for each $i \in N$ we have $\mathbb{E}_{t-1}[e^2_{i,t}] = 1$. One can, then, evaluate deviations from this expectations and define $\psi_{i,t}$ as a volatility shock in the univariate case as follows:

$$\psi_{i,t} \equiv e^2_{i,t} - 1 = \frac{(r_{i,t} - r^f - \beta'_i\mathbf{f}_t)^2 - h_{i,t}}{h_{i,t}} \tag{10}$$

Going back to the object of our study, consider the vector representing the standardized residuals of real exchange rates $\boldsymbol{e}^R_t \equiv (e^R_{1,t}, \ldots, e^R_{N,t})'$ and assume factors are sufficient to reduce the contemporaneous correlations of to zero i.e.,

$$\mathbb{E}_{t-1}(\boldsymbol{e}^R_t\boldsymbol{e}^{R\prime}_t) = \mathbb{I} \tag{11}$$

To obtain the series of standardized residuals, we assume and estimate for each series of real exchange rate returns a single factor model with a first-order auto-regressive term (conditional upon rejecting the null of time independence in the first moment), where the cross-sectional average of returns is used as the single factor, with GARCH$(1,1)$

errors (conditional upon rejecting the null of time independence in the second moment) for simplicity. The average of returns seems to capture most of the correlation between real exchange rates. At it can be seen in Table 2, the average correlation of the raw real exchange returns is 0.300 whereas of standardized residuals of real exchange rates is −0.019. Similarly, we can estimate a factor model with GARCH errors for the nominal exchange rates and relative prices.

Table 2: Average cross-sectional correlations for real exchange rates.

	$\bar{\rho}_{[r_R]}$	$\bar{\rho}_{[\hat{e}_R]}$	$\bar{\rho}_{[\hat{e}^2_R]}$
Correlation	0.300	-0.019	0.049***
GEOVOL test			21.32

*p<0.1; **p<0.05; ***p<0.01

As expected, idiosyncrasies in the the real exchange rates still have correlated volatilities. Assumption (11) implies that the standardized residuals are orthogonal with unit variance. It does not mean however that they are independent. This observation in the time series was the key motivation for the original ARCH model of Engle (1982) and is the key motivation for the GEOVOL model of Engle & Campos-Martins (2020) in the cross section. In fact, the squared standardized residuals of real exchange rates are correlated. Referring back to Table 2, we can see that their average correlation is 0.049, which is positive and statistically significant according to the statistical test proposed by Engle & Campos-Martins (2020). Under the null of a zero average correlation, the test statistic follows a standard normal distribution. For the sample of squared standardized residuals of real exchange rates, the test statistic is 21.32, which can be rejected at the 1% significance level. Because the squares of the standardized residuals are correlated, the co-movements of volatilities are most likely caused by the correlation between shocks to volatility. Figure 2 shows the cross-sectional average of the estimated volatilities for the different sets of variables. Volatilities seem to not only co-move within the same set of variables but also between sets. This co-movement is particularly noticeable between the volatilities of real and nominal exchange rates.

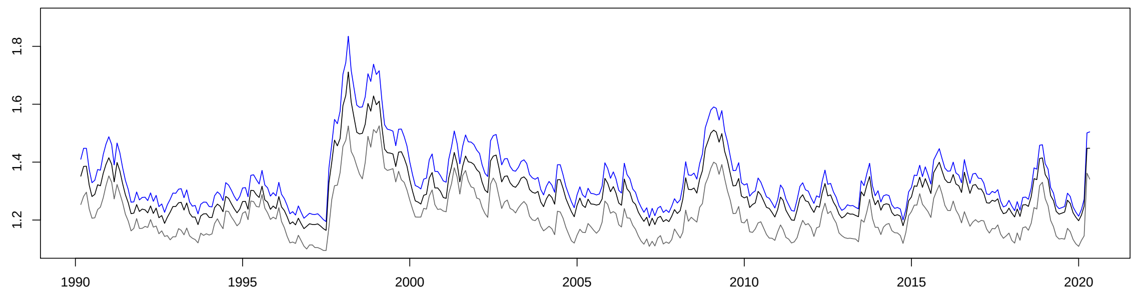

Figure 2: The cross-sectional average of the estimated GARCH-type volatilities of the real (black) and nominal (blue) exchange rates, and of the relative prices (grey).

Define a shock to the ith volatility of real exchange rates as follows

$$\psi_{i,t}^{R} \equiv \left(e_{i,t}^{R}\right)^2 - 1 = \frac{(r_{i,t}^{R} - \alpha_i^R r_{i,t-1}^{R} - \beta_i^R \bar{r}_t^R)^2 - h_{i,t}^{R}}{h_{i,t}^{R}}.$$

The volatility shock $\psi_{i,t}^{R}$ represents the proportional difference between the squared real exchange rate idiosyncrasy and its expectation. To study the determinants of volatility shocks to the real exchange rates, we regress volatility shocks to the real exchange rates ($\psi_{i,t}^{R}$) on the volatility shocks to the nominal exchange rates ($\psi_{i,t}^{E}$) and to the relative prices ($\psi_{i,t}^{P}$). For each currency, we run the regression (by taking the volatility shocks as if they were observed):

$$\psi_{i,t}^{R} = \delta_i^E \psi_{i,t}^{E} + \delta_i^P \psi_{i,t}^{P} + \delta_i^{EP} \psi_{i,t}^{EP} + v_{i,t},$$

where $\psi_{i,t}^{EP} = \psi_{i,t}^{EP} \times \psi_{i,t}^{P}$ is an interaction term. For most currencies, volatility shocks to both the nominal exchange rates and relative prices seem to explain the volatility shocks to the real exchange rates. The average R^2 for the regressions with all three variables is 0.659. With only $\psi_{i,t}^{E}$ as regressor, the average among all regressions is 0.634, which supports the view that the nominal exchange rates component has much higher explanatory power compared to relative prices component of real exchange rates. In terms of the magnitude of the effects (among the statistically significant coefficients), on average, $\bar{\bar{\hat{\delta}}}^{E} = 0.897$, $\bar{\bar{\hat{\delta}}}^{P} = -0.017$ and $\bar{\bar{\hat{\delta}}}^{EP} = 0.076$. Co-movements of real exchange rates seem to mostly arise from volatility shocks to the nominal exchange rates. In other words, shocks affecting nominal exchange rates are the main drivers of simultaneous changes in the volatilities of real exchange rates. Nominal shocks appear to have real effects at the global scale.

5.2 The endogenous model of volatility co-movements

Take the random standardized residuals of real exchange rates, $e^R_{i,t}$ as if they were observed. A data generating process for $e^R_{i,t}$ is assumed from an endogenous volatility factor, denoted by x^R_t, and the random standard normal variables $\varepsilon^R_{i,t}$ as follows

$$e^R_{i,t} = \sqrt{g^R_{i,t}}\varepsilon^R_{i,t} \tag{12}$$

where $g^R_{i,t} \equiv g^R_{i,t}(s^R_i, x^R_t)$ is non-negative for every $t \in [1, T]$ with $\mathbb{E}[g^R_{i,t}(s^R_i, x^R_t)] = 1$, which satisfies $\mathbb{E}[\left(e^R_{i,t}\right)^2] = 1$ for every i. Let x^R_t represent the endogenous volatility factor in the real exchange rates at time t and s^R_i represent the volatility factor loading for asset i, i.e. the fraction of the volatility factor that impacts the ith real exchange rate's volatility. To measure common volatility shocks and model volatility co-movements, we follow Engle & Campos-Martins (2020) and assume

$$g^R_{i,t}(s^R_i, x^R_t) \equiv s^R_i \left(x^R_t - 1\right) + 1, \tag{13}$$

$x^R_t > 0, t = 1, \ldots, T$, and $0 \leq s^R_i \leq 1, i = 1, \ldots, N$, which therefore satisfies 11. In this setting, the realised $\left(e^R_{i,t}\right)^2$ is sometimes bigger than one and on other times smaller than one. When for many assets $\left(e^R_{i,t}\right)^2$ is bigger than one at the same time, this can be interpreted as a common volatility shock to the real exchange rates. On an application to country exchange traded funds, Engle & Campos-Martins (2020) associate such a common shock to geopolitical news due to of its impact on a very wide range of assets, asset classes, and countries.

A simple linear regression of $\hat{x}^R_t - 1$ on $\hat{x}^E_t$, $\hat{x}^P_t$, and $\hat{x}^{EP}_t$, where $\hat{x}^{EP}_t = \hat{x}^E_t \times \hat{x}^P_t$ is an interaction term, allows us to study whether nominal exchange rates and price common shocks, and their interaction, respectively, drive the common volatility shocks to the real exchange rates. By applying a general-to-specific approach (see Pretis et al. (2018)), the estimation results are presented in Table 3. As reported, common shocks to nominal exchange rates materialise into common shocks to the volatilities of real exchange rates in the general (unrestricted) model and only that variable is selected in the final (restricted) model at the 1% significance level.

Table 3: Selecting the drivers of common shocks to the real exchange rates (as measured by $\hat{x}_t^R$).

	General	Specific
$\hat{x}_t^E$	0.347^{***}	0.356^{***}
	(0.025)	(0.025)
$\hat{x}_t^P$	-0.023	
	(0.016)	
$\hat{x}_t^{EP}$	0.015^{*}	
	(0.007)	
Observations	363	363
R^2	0.372	0.363
Residual Std. Error	0.885	0.888

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

The 12-month rolling-window average of the estimated $\hat{x}^R$, $\hat{x}^E$ and $\hat{x}^P$ are plotted in Figure 3. The volatility factor shows high variability for both real and nominal exchange rates. The common shocks to the real exchange rates seem to be almost entirely driven by the common shocks to the nominal exchange rates. Even though at the beginning of the sample there appears to be some similarity between the common shocks to both real exchange rates and the relative prices, during the last two decades (with the exception of the global financial crisis) there seemed to be little co-movement of relative price volatilities.

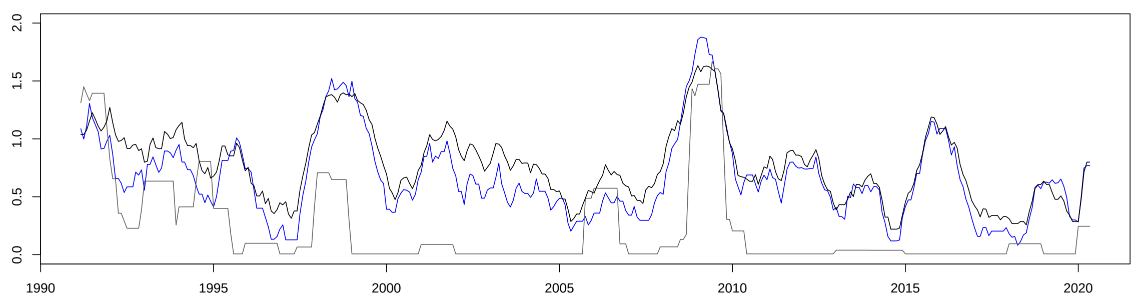

Figure 3: The 12-month rolling window average of the endogenous volatility factor of the real $\hat{x}^R$ (black) and nominal $\hat{x}^E$ (blue) exchange rates, and of the relative prices $\hat{x}^P$ (grey).

Given that the common volatility of real exchange rates is mainly driven by common volatility shocks to the nominal exchange rates, we proceed the analysis by focusing solely on the volatility co-movements of the nominal exchange rates. Assuming model (12), the estimated most extreme common shocks of the volatilities of the nominal exchange rates and their factor loadings are summarized, respectively, in Table 4 and in Table 5. Many common shocks can be easily identified such as those during the COVID-19 pandemic in 2020, the introduction of the euro in 1999 (electronically), the Asian financial crisis in 1997, the global financial crisis in 2008, among others.

Table 4: The most extreme common volatility shocks to the nominal exchange rates.

Date	$\hat{x}_t^E$	$\bar{r}_t^E$
2008-09	9.626	3.988
2020-03	8.781	3.755
1999-01	7.139	0.916
1997-07	7.133	1.983
1993-02	7.100	1.178
2009-02	6.911	3.423
2020-02	6.900	1.773
1991-03	6.242	3.182
2008-02	5.683	−0.727
1991-06	5.680	2.445
2013-05	5.641	0.895
2014-12	5.557	2.346
1998-10	5.262	−2.064
2002-06	4.933	−1.112
2008-12	4.566	−0.897

Different currencies have different volatility factor loadings. This means that currencies with bigger loadings have bigger fractions of the volatility factor affecting their volatilities and so are more exposed to common volatility shocks than others. This gives room for hedging against common shocks, which traditional diversification strategies do not allow. We refer to Engle & Campos-Martins (2020) for the portfolio optimization criterion when in the presence of geopolitical risk.

Table 5: The volatility factor loadings $\hat{s}_i^E$ on the endogenous volatility factor.

AVG	0.434	CZK	0.141
CNY	0.371	PLN	0.136
THB	0.243	TRY	0.119
HUF	0.237	RUB	0.119
EUR	0.236	BRL	0.108
TWD	0.216	NOK	0.105
DKK	0.212	SEK	0.102
SGD	0.211	JPY	0.097
PHP	0.205	GBP	0.088
HKD	0.189	ZAR	0.079
INR	0.185	ILS	0.075
KRW	0.183	AUD	0.075
CHF	0.176	CLP	0.063
MXN	0.159	CAD	0.055
IDR	0.154	NZD	0.021

$\text{AVG}_t = 1/N \sum e_{i,t}^E$

As in Table 2, denote the average empirical correlation across the pairwise correlations of the squared standardized residuals of real exchange rates, $\hat{\boldsymbol{e}}_R^2$, as $\bar{\rho}_{\hat{\boldsymbol{e}}_R^2}$. In Table 6, we summarize $\bar{\rho}_{\hat{\boldsymbol{e}}_R^2}$ for the raw and standardized $\hat{\boldsymbol{e}}_R^2$, namely standardized by the estimated volatility factor of real exchange rates $\hat{\boldsymbol{g}}^R$ (whose elements are defined in (13)), and similarly of nominal exchange rates $\hat{\boldsymbol{g}}^E$, and relative prices $\hat{\boldsymbol{g}}^P$.

Table 6: Average correlation of $\hat{\boldsymbol{e}}_R^2$ for different standardization procedures.

$\bar{\rho}_{\hat{\boldsymbol{e}}_R^2}$	$\bar{\rho}_{(\hat{\boldsymbol{e}}_R^2/\hat{\boldsymbol{g}}^R)}$	$\bar{\rho}_{(\hat{\boldsymbol{e}}_R^2/\hat{\boldsymbol{g}}^E)}$	$\bar{\rho}_{(\hat{\boldsymbol{e}}_R^2/\hat{\boldsymbol{g}}^P)}$
0.049	0.007	0.015	0.051

Comparing the last two columns, we conclude that $\bar{\rho}_{\hat{\boldsymbol{e}}_R^2}$ can be significantly reduced when $\hat{\boldsymbol{e}}_R^2$ are standardized by the estimated volatility factors of nominal exchange rates, $\hat{\boldsymbol{g}}^E$ (rather than relative prices i.e., $\hat{\boldsymbol{g}}^P$). This means that common shocks to the nominal exchange rates are the main drivers of volatility co-movements of real exchange rates, which further supports our previous results. Let's now turn the attention to what then drives volatility co-movements in the nominal exchange rates.

5.3 The model with exogenous volatility factors

In order to also include exogenous information in the volatility factor model, we use interest rate differentials (with respect to the U.S. interest rate) and inflation differentials (with respect to the U.S. inflation rate). We take first-differences to compute both interest and inflation volatility shocks. The standardized residuals are obtained by regressing the returns on not only the return cross-sectional average (proxy for market factor) but also the interest rate and inflation differentials (after computing their first-differences). On average, their effects on returns are, respectively, 0.994 −0.056 and 0.433. The coefficient associated to the return cross-sectional average is highly significant for all nominal exchange rates. Neither the interest rate nor the inflation differentials seem to affect nominal exchange rate returns as many of coefficients are not statistically different from zero.

We can specify a multiplicative function $g_{i,t}^E$ that is a data generating process for the squared standardized residuals of the nominal exchange rates, $e_{i,t}^E$, from the endogenous volatility factor x_t^G (and s_i^G, where the superscript G is introduced to distinguish it from the other factors), the exogenous volatility factors x_t^i, x_t^π (and s_i^i, s_i^π), which are assumed as observed, and $\epsilon_{i,t}$ as follows:

$$e_{i,t}^E = \sqrt{g_{i,t}^E}\epsilon_{i,t}^E,$$

where $g_{i,t}^E$ can be specified as either

$$g_{i,t,(14)}^E \equiv \left[s_{i,(14)}^G \left(x_{t,(14)}^G - 1\right) + 1\right] \tag{14}$$

$$g_{i,t,(15)}^E \equiv \left[s_{i,(15)}^G \left(x_{t,(15)}^G - 1\right) + 1\right] \times \left[s_{i,(15)}^i \left(x_t^i - 1\right) + 1\right] \tag{15}$$

$$g_{i,t,(16)}^E \equiv \left[s_{i,(16)}^G \left(x_{t,(16)}^G - 1\right) + 1\right] \times \left[s_{i,(16)}^i \left(x_t^i - 1\right) + 1\right] \times \left[s_{i,(16)}^\pi \left(x_t^\pi - 1\right) + 1\right]. \tag{16}$$

Other specifications are certainly possible as long as $g_{i,t}^E, i = 1, \ldots, n$, is non-negative with expected value 1 such that (11) is satisfied. For comparison, the exogenous volatility factors $\hat{x}_t^i$ and $\hat{x}_t^\pi$ are depicted in Figure 4, alongside $x_{t,(14)}^G$.

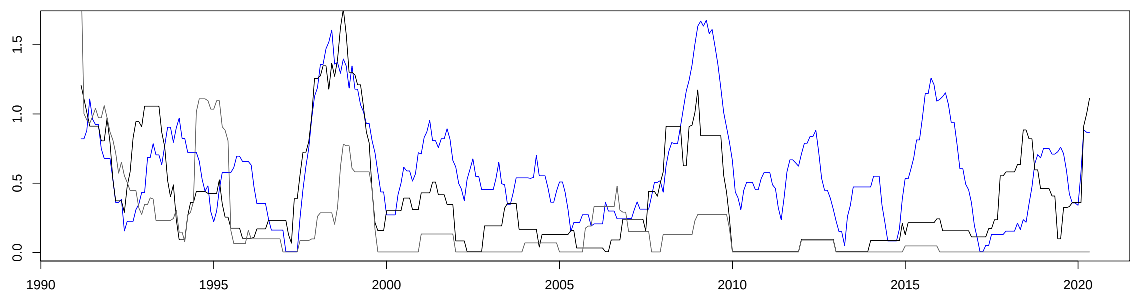

Figure 4: The 12-month rolling window average of the volatility global factor $\hat{x}^G_{(14)}$ (blue), $\hat{x}^i_t$ (black), and $\hat{x}^\pi_t$ (grey).

We proceed to the estimation of the volatility factor models (14)-(16). The estimated most extreme common volatility shocks and the volatility factor loadings (for both endogenous and exogenous factors) in Appendix D. The 12-month average of the endogenous volatility factor for all models are depicted in Figure 5. Results point out a similar trajectory across all models, either including or excluding exogenous factors. Nevertheless, some differences are noticeable around crisis periods such as the global financial crisis, and in more recent years.

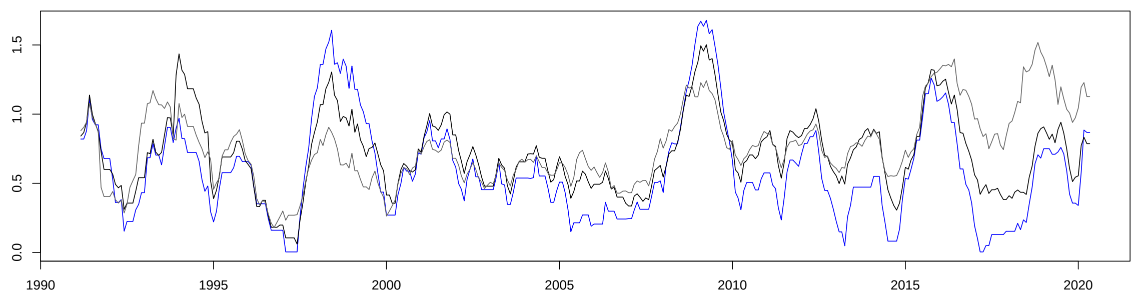

Figure 5: The 12-month rolling window average of the endogenous volatility global factor from different specifications: no exogenous factors, $\hat{x}^G_{(14)}$ (blue), with exogenous factor $x^i_{(15)}$, $\hat{x}^G_{(15)}$ (black), and with exogenous factors x^i and x^π, $\hat{x}^G_{(16)}$ (grey).

The correlations between the estimated endogenous volatility factors $\hat{x}^G_{(.)}$ depicted in Figure 5 are summarised below.

Table 7: Correlations between the endogenous volatility factor $\hat{x}^G_{(.)}$ obtained from different specifications.

	$\hat{x}^G_{(14)}$	$\hat{x}^G_{(15)}$	$\hat{x}^G_{(16)}$
$\hat{x}^G_{(14)}$	1	0.885	0.501
$\hat{x}^G_{(15)}$	0.885	1	0.670
$\hat{x}^G_{(16)}$	0.501	0.670	1

Denote the average correlation of the squared standardized residuals of nominal exchange rates $\hat{\boldsymbol{e}}^2_E$ as $\bar{\rho}_{\hat{\boldsymbol{e}}^2_E}$. In Table 8, we summarize $\bar{\rho}_{\hat{\boldsymbol{e}}^2_E}$ for their standardized by the endogenous and composite (endogenous and exogenous) volatility factors counterparts.

Table 8: Average correlation of $\hat{\boldsymbol{e}}^2_E$ for different standardization, where $\bar{\rho}_{\hat{\boldsymbol{e}}^2_E} = 0.046$.

$\bar{\rho}_{[\hat{\boldsymbol{e}}^2_E/\hat{\boldsymbol{g}}^E_{(14)}]}$	
0.003	
$\bar{\rho}_{[\hat{\boldsymbol{e}}^2_E/\hat{\boldsymbol{g}}^G_{(15)}]}$	$\bar{\rho}_{[\hat{\boldsymbol{e}}^2_E/\hat{\boldsymbol{g}}^E_{(15)}]}$
0.021	**0.001**
$\bar{\rho}_{[\hat{\boldsymbol{e}}^2_E/\hat{\boldsymbol{g}}^G_{(16)}]}$	$\bar{\rho}_{[\hat{\boldsymbol{e}}^2_E/\hat{\boldsymbol{g}}^E_{(16)}]}$
0.036	0.016

A comparison of the performance in capturing common volatility shocks between the different models reveals that $\bar{\rho}_{\hat{\boldsymbol{e}}^2_E}$ is minimised for $\hat{\boldsymbol{e}}^2_E$ standardized by the estimated composite volatility factor where an endogenous factor and an exogenous factor measuring common volatility shocks to the interest rate differentials are both included i.e., by $\hat{\boldsymbol{g}}^E_{(15)}$. Volatility co-movements of nominal exchange rates seem to be explained by direct common volatility shocks to the nominal exchange rates, and indirectly by common volatility shocks to the interest rate differentials.

5.4 Discussion of results

The results presented in this section show that the endogenous common volatility shocks model of Engle & Campos-Martins (2020) performs well in modelling the cross-sectional correlation in real exchange rates volatilities (Table 6). Moreover, a significant amount of evidence suggests that this cross-sectional correlation in real exchange rates is, as expected, a result of common volatility shocks to nominal exchange rates rather than price differentials. As the return of real exchange rates are simply a linear combination of the returns of nominal exchange rates and price differentials, we decided to focus the analysis on the exogenous drivers of the common volatility shocks to nominal exchange rates.

In Section 5.3, we present a framework that allows us to study how exogenous factors may drive this common volatility shocks to nominal exchange rates. In fact, this is a general framework and can be considered as an extension of the model of Engle & Campos-Martins (2020) to allow for exogenous drivers of common volatility shocks. In order to introduce our framework, we consider the two major drivers of nominal exchange rates fluctuations as exogenous variables: interest rate differentials and inflation differentials. The results from Section 5.3 show common shocks to interest rate differentials as a key driver of common volatility shocks in nominal exchange rates. The model with both the endogenous term x_t^G and the interest rate differential term x_t^i performs significantly better than all other models considered in purging the cross-sectional correlation left in the volatilities of nominal exchange rates. These results provide an interesting link between the sources of the remaining part of the PPP Puzzle and another major topic of study in empirical international finance: the Interest Rate Parity.

6 Concluding Remarks

Most of the discussion about the Purchasing Power Parity (PPP) Puzzle of Rogoff (1996) has pertained to the reversion speed of deviations from PPP. Much less attention, however, has been given to the other component of the puzzle: the high volatilities of real exchange rates. In this paper, we use latest developments from the financial econometrics literature in second moments dynamics to provide a framework that is capable of explaining the econometric sources of these volatilities and further provides a framework to link these to their possible macro-financial drivers.

In Section 4, we present an econometric framework based on the Cross-Sectionally Augmented Autoregressive Distributed Lag (CR-ARDL) model of Pesaran (2006) for the decomposition of real exchange rate volatility into its building blocks. As discussed by Taylor (2002), Taylor & Taylor (2004), Ganguly & Breuer (2010) and many others, understanding the drivers of real exchange rate volatilities in the short-to-medium-run is the key remaining part of the PPP Puzzle. Our estimation results from the decomposition show that this task can be re-framed. If one seeks to understand the sources of high real exchange rate volatility – and hence understand the remaining part of the PPP Puzzle – one must address the cross sectional correlation in volatilities across currencies.

Inspired by the results from Section 4 regarding the importance of cross-sectional correlations in explaining real exchange rate volatilities, in Section 5 we propose an econometric model based on the endogenous common volatility shocks framework of Engle & Campos-Martins (2020) to model the dynamics of real exchange rate volatilities. The proposed framework presents encouraging results when modelling these cross-sectional correlations. It successfully gives structure to these common volatility dependencies in real exchange rates and further allows us to propose an extension to the framework in order to study the impact of other macro-financial variables on this common volatility movements. In fact, this is extension is a general framework and can be considered as an extension of the model of Engle & Campos-Martins (2020) to allow for exogenous drivers of common volatility shocks. The results from our proposed extension show common shocks to interest rate differentials as a key driver of common volatility shocks in exchange rates, hence building a bridge between our analysis of the PPP Puzzle and the Interest Rate Parity literature.

There is a vast body of research regarding the links between interest rate differentials and exchange rate dynamics. However, a topic that is particularly interested in studying the relationship between these two variables is the study of the Interest Rate Parity[8]. The failure of the Interest Rate Parity in providing useful guidance to exchange rate behaviour has, in fact, been a topic of intense study[9] and the source of another puzzle in the international finance literature, known as the Forward Premium Puzzle. Although the Forward Premium Puzzle of Fama (1984) refers to futures of exchange rates, we find it noteworthy the link between these two variables presented in our results as being, at least partially, the source of the PPP Puzzle. An interesting way forward in both empirical and theoretical international finance would be to study how common volatility shocks, shown to be the main source of the PPP Puzzle, also affect Interest Rate Parity and hence the findings that originated the Forward Premium Puzzle.

This research expects to contribute to a better understanding of the PPP Puzzle. More specifically, to the question of why real exchange rates are so volatile in the sort-to-medium run. Our application of the exogenous drivers extension to the endogenous common volatility shocks framework of Engle & Campos-Martins (2020) was limited to the two series which are more meaningfully related to exchange rate dynamics according to the literature. Further research could study other drivers of common shocks to exchange rate volatilities in a more holistic approach, diving into even higher frequencies by considering daily financial series. Future studies could, more broadly, seek to understand exogenous drivers of common volatility shocks in other asset classes by re-framing the framework presented in Section 5. Engle & Campos-Martins (2020) show that international equity markets present a similar behavior regarding common volatility shocks. This feature in multiple asset classes suggests an avenue to explored linking these common volatility shocks to possible exogenous drivers.

[8]For an intro and review of Interest Rate Parity see Stein (1962), Glahe (1967), Aliber (1973), Wu & Chen (1998), amongst others.

[9]See Fama (1984) for original Forward Premium Puzzle and Bansal (1997), Bansal & Dahlquist (2000), Burnside et al. (2009), and others for more recent interpretations.

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A List of Countries and Market Classification

Table 9: List of countries in the dataset and respective MSCI (2020) market classification

Currency	Country	MSCI Market Classification
AUD	Australia	Developed Market
BRL	Brazil	Emerging Market
CAD	Canada	Developed Market
CHF	Switzerland	Developed Market
CLP	Chile	Emerging Market
CNY	China	Emerging Market
CZK	Czech Republic	Emerging Market
DKK	Denmark	Developed Market
EUR	Euro Area	Developed Market
GBP	United Kingdom	Developed Market
HKD	Hong Kong	Developed Market
HUF	Hungary	Emerging Market
IDR	Indonesia	Emerging Market
ILS	Israel	Developed Market
INR	India	Emerging Market
JPY	Japan	Developed Market
KRW	South Korea	Emerging Market
MXN	Mexico	Emerging Market
NOK	Norway	Developed Market
NZD	New Zealand	Developed Market
PHP	Philippines	Emerging Market
PLN	Poland	Emerging Market
RUB	Russia	Emerging Market
SEK	Sweden	Developed Market
SGD	Singapore	Developed Market
THB	Thailand	Emerging Market
TRY	Turkey	Emerging Market
TWD	Taiwan	Emerging Market
ZAR	South Africa	Emerging Market

B Volatility Models for Real Exchange Rates

B.1 TARCH Model

We estimate a TARCH(P,O,Q) model of Zakoian (1994) with the following model specification:

$$
\begin{aligned}
r_t &= \epsilon_t \\
\epsilon_t &= \sigma_t e_t \\
\sigma_t &= \omega + \sum_{p=1}^{P} \alpha_p |\epsilon_{t-p}| + \sum_{o=1}^{O} \gamma_o |\epsilon_{t-o}| 1_{[\epsilon_{t-o}<0]} + \sum_{q=1}^{Q} \beta_q \sigma_{t-q} \\
e_t &\overset{i.i.d.}{\sim} N(0,1)
\end{aligned}
$$

B.2 GJR-GARCH Model

We estimate a GJR-GARCH(P,O,Q) model of Glosten et al. (1993) with the following model specification:

$$
\begin{aligned}
r_t &= \epsilon_t \\
\epsilon_t &= \sigma_t e_t \\
\sigma_t^2 &= \omega + \sum_{p=1}^{P} \alpha_p \epsilon_{t-p}^2 + \sum_{o=1}^{O} \gamma_o \epsilon_{t-o}^2 1_{[\epsilon_{t-o}<0]} + \sum_{q=1}^{Q} \beta_q \sigma_{t-q}^2 \\
e_t &\overset{i.i.d.}{\sim} N(0,1)
\end{aligned}
$$

B.3 EGARCH Model

We estimate a EGARCH(P,O,Q) model of of Nelson (1991) with the following model specification:

$$
\begin{aligned}
r_t &= \epsilon_t \\
\epsilon_t &= \sigma_t e_t \\
ln(\sigma_t^2) &= \omega + \sum_{p=1}^{P} \alpha_p \left(\left| \frac{e_{t-p}}{\sigma_{t-p}} \right| - \sqrt{\frac{2}{\pi}} \right) + \sum_{o=1}^{O} \gamma_o \frac{e_{t-o}}{\sigma_{t-o}} + \sum_{q=1}^{Q} \beta_q ln(\sigma_{t-q}^2) \\
e_t &\overset{i.i.d.}{\sim} N(0,1)
\end{aligned}
$$

B.4 Selected models

Table 10: Real exchange rate volatility model selected for each currency

Currency	Country	Model Selected
AUD	Australia	GJR-GARCH(1,0,1)
BRL	Brazil	EGARCH(2,1,1)
CAD	Canada	TARCH(1,0,0)
CHF	Switzerland	EGARCH(0,1,1)
CLP	Chile	EGARCH(1,1,1)
CNY	China	EGARCH(2,0,1)
CZK	Czech Republic	TARCH(1,0,1)
DKK	Denmark	EGARCH(1,0,1)
EUR	Euro Area	GJR-GARCH(0,1,2)
GBP	United Kingdom	GJR-GARCH(1,0,1)
HKD	Hong Kong	GJR-GARCH(1,1,1)
HUF	Hungary	TARCH(1,0,1)
IDR	Indonesia	EGARCH(1,0,1)
ILS	Israel	GJR-GARCH(1,0,1)
INR	India	TARCH(1,0,1)
JPY	Japan	GJR-GARCH(1,0,0)
KRW	South Korea	EGARCH(2,1,1)
MXN	Mexico	EGARCH(2,1,1)
NOK	Norway	GJR-GARCH(1,0,0)
NZD	New Zealand	GJR-GARCH(1,0,1)
PHP	Philippines	GJR-GARCH(1,1,1)
PLN	Poland	GJR-GARCH(1,0,1)
RUB	Russia	EGARCH(1,1,2)
SEK	Sweden	GJR-GARCH(1,0,0)
SGD	Singapore	GJR-GARCH(1,0,1)
THB	Thailand	GJR-GARCH(1,0,1)
TRY	Turkey	TARCH(1,1,0)
TWD	Taiwan	EGARCH(1,0,1)
ZAR	South Africa	EGARCH(2,2,1)

C Covariance Model for Nominal Exchange Rate and Price Differentials

C.1 The AG-DCC Multivariate GARCH Model

We begin by defining:

$$\begin{aligned} \boldsymbol{r}_t &= \boldsymbol{\epsilon}_t \\ \boldsymbol{\epsilon}_t &= \boldsymbol{\Sigma}_t^{1/2}\boldsymbol{e}_t \\ \boldsymbol{e}_t &\overset{i.i.d.}{\sim} N(\boldsymbol{0}, \boldsymbol{I}_2) \end{aligned}$$

And then we model $\boldsymbol{\Sigma}_t$ according to the AG-DCC GARCH(M,L,N) specification of Cappiello et al. (2006):

$$\begin{aligned} \boldsymbol{\Sigma}_t &= \boldsymbol{D}_t\boldsymbol{P}_t\boldsymbol{D}_t \\ \boldsymbol{P}_t &= \boldsymbol{Q}_t^*\boldsymbol{Q}_t\boldsymbol{Q}_t^* \\ \boldsymbol{Q}_t &= (\bar{\boldsymbol{P}} - \sum_{m=1}^{M}\boldsymbol{A}_m'\bar{\boldsymbol{P}}\boldsymbol{A}_m - \sum_{l=1}^{L}\boldsymbol{G}_l'\bar{\boldsymbol{N}}\boldsymbol{G}_l - \sum_{n=1}^{N}\boldsymbol{B}_n'\bar{\boldsymbol{P}}\boldsymbol{B}_n) + \sum_{m=1}^{M}\boldsymbol{A}_m'\boldsymbol{e}_{t-m}\boldsymbol{e}_{t-m}'\boldsymbol{A}_m \\ &\quad + \sum_{l=1}^{L}\boldsymbol{G}_l'\boldsymbol{n}_{t-l}\boldsymbol{n}_{t-l}'\boldsymbol{G}_l + \sum_{n=1}^{N}\boldsymbol{B}_n'\boldsymbol{Q}_{t-n}\boldsymbol{B}_n \\ \boldsymbol{Q}_t^* &= (\boldsymbol{Q}_t \odot \boldsymbol{I}_2)^{\frac{1}{2}} \end{aligned}$$

Where $\boldsymbol{D}_t$ is a diagonal matrix of conditional standard deviations and $\boldsymbol{P}_t$ is the correlation matrix with diagonal one.

C.2 Selected models

Table 11: AG-DCC GARCH(M,L,N) model selected for each currency

Currency	Country	Model Selected
AUD	Australia	AG-DCC(1,0,1)
BRL	Brazil	AG-DCC(1,0,1)
CAD	Canada	AG-DCC(1,0,1)
CHF	Switzerland	AG-DCC(1,0,1)
CLP	Chile	AG-DCC(1,0,1)
CNY	China	AG-DCC(1,0,1)
CZK	Czech Republic	AG-DCC(1,0,1)
DKK	Denmark	AG-DCC(1,0,0)
EUR	Euro Area	AG-DCC(1,1,0)
GBP	United Kingdom	AG-DCC(1,0,1)
HKD	Hong Kong	AG-DCC(1,0,1)
HUF	Hungary	AG-DCC(1,0,1)
IDR	Indonesia	AG-DCC(1,0,1)
ILS	Israel	AG-DCC(1,0,0)
INR	India	AG-DCC(1,0,1)
JPY	Japan	AG-DCC(1,0,0)
KRW	South Korea	AG-DCC(1,0,1)
MXN	Mexico	AG-DCC(2,0,0)
NOK	Norway	AG-DCC(1,0,1)
NZD	New Zealand	AG-DCC(1,0,1)
PHP	Philippines	AG-DCC(1,0,2)
PLN	Poland	AG-DCC(1,0,1)
RUB	Russia	AG-DCC(1,1,1)
SEK	Sweden	AG-DCC(1,0,0)
SGD	Singapore	AG-DCC(1,0,1)
THB	Thailand	AG-DCC(1,0,1)
TRY	Turkey	AG-DCC(1,0,0)
TWD	Taiwan	AG-DCC(1,0,1)
ZAR	South Africa	AG-DCC(1,0,0)

D Estimation results with exogenous volatility factors

D.1 Volatility factor model (15)

Table 12: The most extreme common volatility shocks to the nominal exchange rates with x^i as an exogenous volatility factor.

Date	$\hat{x}^G$	$\bar{r}^E$
1993-12	27.946	0.643
1993-02	7.029	1.178
2020-02	6.950	1.773
1990-11	6.394	0.795
2008-09	6.187	3.988
1999-01	5.630	0.916
2009-02	5.604	3.423
1991-06	5.521	2.445
2015-07	4.948	1.769
2013-05	4.916	0.895
1992-11	4.686	3.658
2011-08	4.585	0.788
2014-12	4.527	2.346
1997-07	4.041	1.983
2015-08	3.962	2.203

Table 13: The volatility factor loadings on x^G.

CNYn	0.319	RUBn	0.158
HKDn	0.288	GBPn	0.157
HUFn	0.254	BRLn	0.144
PHPn	0.251	NOKn	0.135
SGDn	0.245	CADn	0.127
THBn	0.239	SEKn	0.122
DKKn	0.233	MXNn	0.121
PLNn	0.229	TWDn	0.120
INRn	0.227	CHFn	0.118
EURn	0.220	JPYn	0.117
IDRn	0.197	AUDn	0.093
TRYn	0.193	NZDn	0.064
KRWn	0.188	ILSn	0.039
AVGn	0.177	CLPn	0.000
CZKn	0.176	ZARn	0.000

Table 14: The volatility factor loadings on x^i.

CNY	0.848	PHP	0.055
RUB	0.253	PLN	0.048
THB	0.221	NOK	0.046
IDR	0.165	CHF	0.046
MXN	0.125	SEK	0.036
DKK	0.125	AUD	0.035
AVG	0.119	SGD	0.033
JPY	0.119	NZD	0.027
TWD	0.112	GBP	0.022
HUF	0.112	CZK	0.020
ZAR	0.111	CLP	0.000
ILS	0.104	EUR	0.000
BRL	0.086	HKD	0.000
CAD	0.082	INR	0.000
KRW	0.056	TRY	0.000